Measuring the Spins of Rubidium Nuclei by Optical Pumping

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When optical radiation of 795 nm is absorbed by a rubidium atom in its ground state, it undergoes a D_1 transition to its first excited state. Circular-polarized incident light allows the rubidium atom to transition across Zeeman levels, which results in the absorption of the photon. By introducing an external magnetic field, one can vary the energy difference between Zeeman levels. In the experiment, the dependence of the energy difference between Zeeman levels on the external magnetic field is used to determine the nuclear spins of ^{85}Rb and ^{87}Rb. The experiment shows that the nuclear spin of ^{85}Rb is 5/2, and the nuclear spin of ^{87}Rb is 3/2. Additionally, the effect of cell temperature on Rb absorption of resonance frequency radiation is used to determine the absorption cross-section area. The cross-section area of absorption is determined to be 120 ± 8 nm^2.

I. INTRODUCTION

Optical pumping, the process by which light is used to excite atoms to achieve population inversion, is a practical and well-studied marriage between optics and atomic physics. In this experiment, rubidium is pumped to one of its excited states by the absorption of photons with precisely the energy required to drive the atoms from their $^2S_{1/2}$ state to their $^2P_{1/2}$ state via the D_1 transition[1]. The D_1 transition is described in further detail in Sec. II A.

II. NUCLEAR SPIN MEASUREMENT

Rubidium has two naturally occurring isotopes, ^{87}Rb and ^{85}Rb. The goal of this experiment is to determine the nuclear spins of these two isotopes. This is not a straightforward task, as the nuclear spin is coupled with the angular momentum of the single valence electron of rubidium. Nonetheless, it is possible to measure the interaction of these angular momenta with an external magnetic field and quantify the nuclear spins of both isotopes.

A. Theory

Rubidium is an alkali element with a single valence electron. The atom is excited by manipulating the angular momentum of its valence electron. The electron itself has a spin $S = 1/2$. In the ground state, the valence electron occupies the s-orbital, and its angular momentum L is 0. In the first excited state of rubidium, the angular momentum is increased and the electron occupies a p-orbital, corresponding to $L = 1$. If L is non-zero, the angular momentum interacts with the electron spin, and the vector describing the total angular momentum $\mathbf{J} = \mathbf{L} + \mathbf{S}$ can have one of two values, $L+S$ or $L-S$. The D_1 transition mentioned in Sec. I corresponds to a transition from the ground state of Rb to the first excited state labeled $^2P_{1/2}$, with total angular momentum $J = L - S = 1/2$. The difference in energy between these two states is equal to the energy of a photon with a frequency of about 377 THz[3]; with light of this frequency, it is possible to drive the D_1 transition.

The difference between ^{85}Rb and ^{87}Rb that is exploited in this experiment is in their nuclear spins, denoted with the variable I. The total angular momentum J couples with the spin of the nucleus I to produce yet another quantum number $\mathbf{F} = \mathbf{J} + \mathbf{I}$ that labels which hyperfine state the atom occupies. In the ground state, $J = 1/2$ and so F can take on the value of $I + J$ or $I - J$. With no external magnetic field each hyperfine level consists of $2F + 1$ degenerate energy levels, which are labeled by the quantum number M_F ranging from $-F$ to F in integer steps[1]. In the presence of an external magnetic field, the energy of each Zeeman level increases by

$$E_Z = g_F \mu_B B M_F \tag{1}$$

where μ_B is the Bohr magneton, and g_F is an expression of the quantum numbers discussed above[1]:

$$g_F = g_J \frac{F(F+1) + J(J+1) - I(I+1)}{2F(F+1)} \tag{2}$$

and

$$g_J = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} \tag{3}$$

If the incident light is circular-polarized, the D_1 transition necessarily increases or decreases the value of M_F. Right-circular polarized (RCP) light will increase M_F by 1, while left-circular polarized (LCP) light will decrease M_F by 1[1]. Let us assume we induce the D_1 transition with RCP light. Each time a photon is absorbed, it will drive the electron to a higher Zeeman level, while the average decay event will have no effect on M_F. So, after a series of excitements and decays, M_F will be driven to its maximal value, F. Since F is the same in the ground state and in the $^2P_{1/2}$ state after the D_1 transition, no RCP light can be absorbed by an electron in the ground state when $M_F = F$. Thus, the intensity of the RCP light that propagates through the rubidium cell without being scattered is higher when there are more atoms in a *dark state* where $M_F = F$. By introducing RF radiation with precisely the energy between adjacent Zeeman levels, the atoms can transfer between Zeeman states within the same hyperfine level. This allows for atoms in a dark state to transition into a state where $M_F \neq F$ and absorb a photon, decreasing the intensity of the transmitted light.

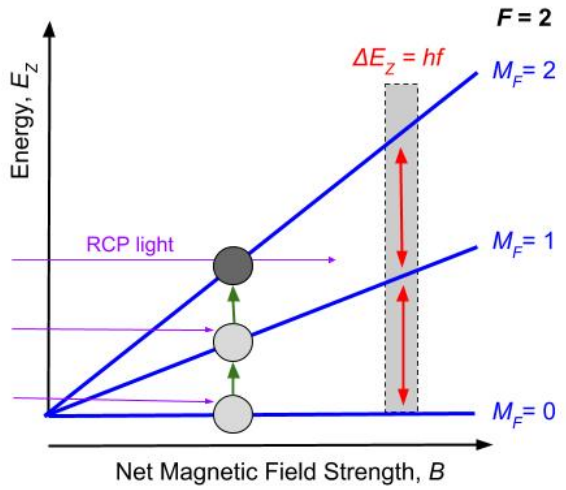

FIG. 1. Green arrows represent the D_1 transition with $\Delta M_F = 1$ followed by a decay with $\Delta M_F = 0$. RCP light can only be absorbed by the atom if $M_F \neq F$, otherwise the light is transmitted. RF radiation with frequency f can drive atoms out of a dark state at a particular magnetic field strength (where $\Delta E_Z = hf$). Note that the Zeeman states with $M_F = -1$ and $M_F = -2$ are not depicted in this diagram.

B. Procedure

The apparatus used in this experiment includes a cell of rubidium vapour (at 50°C) that contains both ^{85}Rb and ^{87}Rb. Circular-polarized light at 377 THz passes horizontally through the cell, and the transmitted light is observed with an optical detector. One can apply magnetic fields to the cell in the vertical direction and in the direction of light propagation. To measure the effect of the earth's magnetic field on the Rb cell, the horizontal field is continuously increased from 0 to 70 μT. Meanwhile, the vertical magnetic field is adjusted to minimize the amount of variance in the location of the dip depicted in Fig. 2. The detector voltage in Fig. 2 dips when the net external magnetic field strength is nearly 0, making the Zeeman levels degenerate; this occurs when the applied magnetic field $B = 15.9 \pm 0.1$ μT, which compensates for the Earth's magnetic field in the opposite direction. Adjusting the vertical field to minimize the dip's variance effectively cancels the vertical component of the Earth's magnetic field.

Next, RF radiation with an amplitude of 2 V is applied to the Rb cell. The frequency of the radiation is varied from 100 kHz to 220 kHz in 20 kHz increments. When continuously increasing the horizontal magnetic field, two distinct dips emerge in the detector voltage that occur in addition to the dip at 15.9 μT. An example of this can be seen in Fig. 3. When the energy between Zeeman levels is equal to the energy of the RF radiation, atoms in a dark ground state can transfer between Zeeman states within the same hyperfine level and undergo the D_1 transition. The net magnetic field strengths that produce high amounts of photon absorption are recorded in Fig. 4.

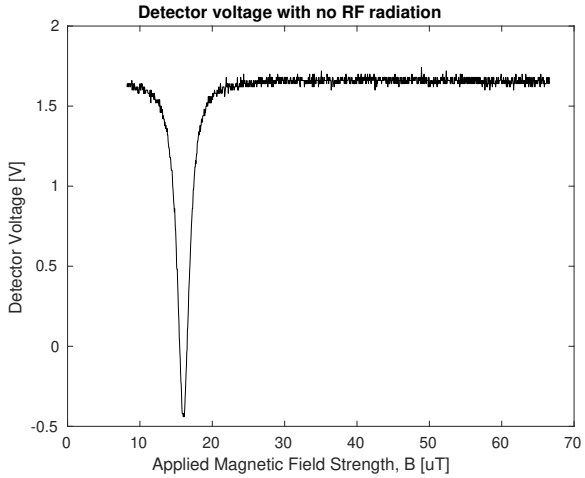

FIG. 2. Amount of transmitted light corresponding to various applied magnetic fields. At $B = 15.9$ μT, there is a significant dip in the amount of transmitted light. This applied magnetic field cancels the Earth's magnetic field and makes the Zeeman levels degenerate, scattering the Zeeman states and allowing for more absorption.

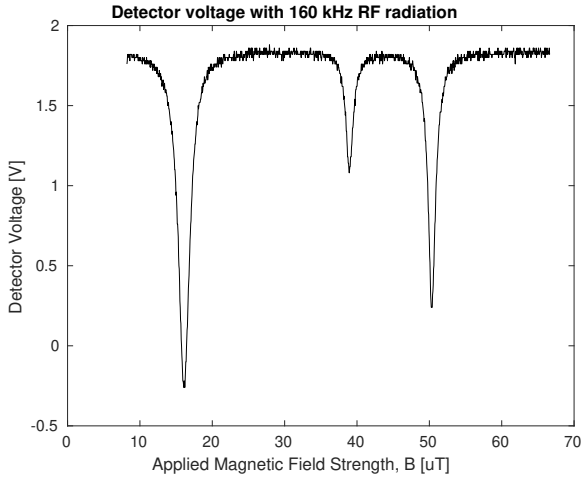

FIG. 3. 160 kHz radiation is applied to the Rb cell. There is a large amount of photon absorption when the applied magnetic field $B = 38.9$ μT and also when $B = 50.4$ μT. The peaks in absorption occur when adjacent Zeeman levels have an energy difference of 160 kHz. The magnetic field strength required to drive the Zeeman levels 160 kHz apart is different for each isotope, and so we see high absorption at precisely these magnetic field strengths.

High Amplitude RF

100 kHz RF radiation with an amplitude of 5 V is applied to the Rb cell. There are additional magnetic field strengths that result in high amounts of absorption, which can be seen in Fig. 5. These additional magnetic field strengths separate the Zeeman levels by 200 kHz; this suggests that if the amplitude of the RF radiation is sufficiently large, it can cause transitions between energy levels separated by twice the energy of the RF radiation.

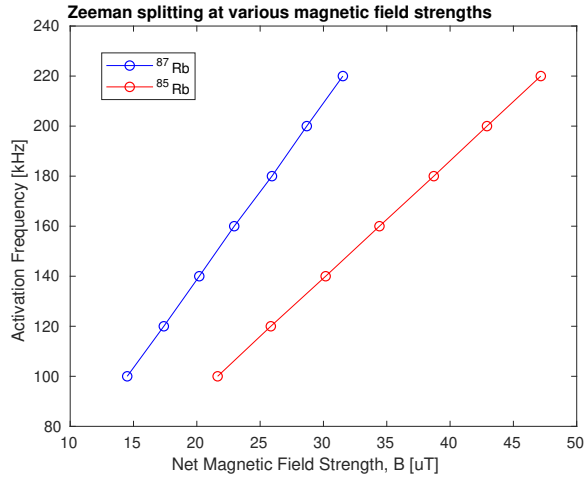

FIG. 4. The energy difference between adjacent Zeeman levels at various magnetic field strengths for ^{87}Rb and ^{85}Rb in their ground states. Obtained by measuring the net magnetic field strengths that result in high absorption given RF radiation at various frequencies.

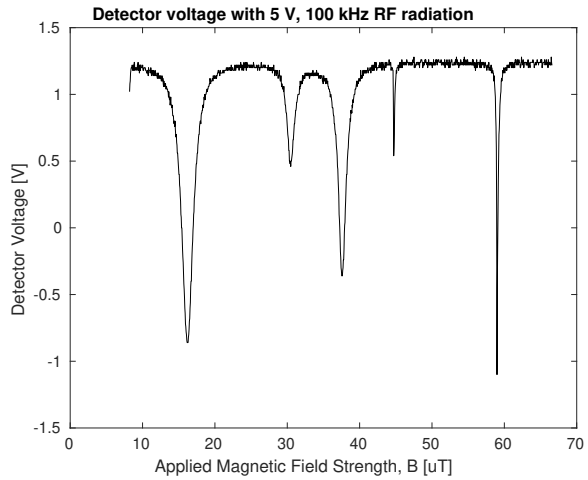

FIG. 5. 100 kHz radiation with amplitude increased to 5 V is applied to the cell. The two rightmost dips in the amount of light transmitted through the cell correspond to magnetic field strengths that separate Zeeman levels by twice the energy of the radiation.

C. Results

From Fig. 4, it can be seen that the relationship between net magnetic field strength B and the energy difference between Zeeman levels $\Delta E_Z = h\Delta f$ is highly linear. The proportionality constant between B and ΔE_Z is different for the two isotopes due to the difference in their nuclear spins. Eqn. 1 can be written in terms of ΔE_Z:

$$\frac{\Delta f}{B} = \frac{\Delta E_Z}{hB} = \frac{g_F \mu_B}{h} \tag{4}$$

The value of $\Delta f/B$ for each isotope can be taken from a linear fit of the curves in Fig. 4. For ^{85}Rb, we have,

$$\left(\frac{\Delta f}{B}\right)_{^{87}\mathrm{Rb}} = (7.05 \pm 0.06)\ \mathrm{GHz/T} \tag{5}$$

and for ^{87}Rb,

$$\left(\frac{\Delta f}{B}\right)_{^{85}\mathrm{Rb}} = (4.70 \pm 0.02)\ \mathrm{GHz/T} \tag{6}$$

By multiplying these values by h/μ_B, it is possible to extract g_F for each isotope.

	^{87}Rb	^{85}Rb
g_F (Measured)	0.504±0.004	0.335±0.001
g_F (Theoretical)	1/2	1/3

The theoretical values of g_F come from Eqn. 2. In the $^2S_{1/2}$ state, $S = 1/2$, $L = 0$, $J = 1/2$, and $F = I \pm 1/2$. Eqn. 2 simplifies to

$$g_F = \pm \frac{1}{I + 0.5} \tag{7}$$

Since $2F + 1$ (the number of Zeeman states) is an integer, I must be of the form $n/2$ with $n \in \mathbb{N}$. Thus g_F is of the form

$$g_F = \pm \frac{2}{n+1},\ n \in \mathbb{N} \tag{8}$$

Indeed, the measured values of g_F are closest to the permissible values 1/2 and 1/3, corresponding to $I = 3/2$ and $I = 5/2$ for ^{87}Rb and ^{85}Rb respectively. The values of I predicted in this experiment match those given in literature[3,4].

III. EFFECT OF CELL TEMPERATURE ON ABSORPTION

The relationship between the temperature of the Rb cell and the atomic density can be exploited to determine the cross-section area of photon absorption.

A. Theory

The relationship between the Temperature and Density is tabulated below[2]:

Temperature (K)	Density (Atoms/m^3 $\times 10^{18}$)
310	0.03
320	0.08
330	0.18
340	0.43
350	0.83
360	1.5
370	3.7
380	6.3
390	12

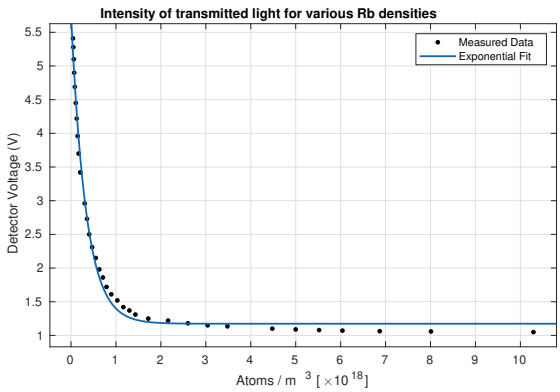

FIG. 6. The intensity of the transmitted light varies with the density of the rubidium according to a decaying exponential function. The black points are determined by measuring the voltage of the detector at different temperatures and calculating the theoretical density at each temperature. The blue curve is an exponential fit that decays to the DC offset of the detector voltage.

The cross-section area of absorption can be inferred by observing the amount of unpolarized light (at 377 THz) that is transmitted through the cell. The relationship between the intensity of the transmitted light I and the density of rubidium ρ is a function of the volume of absorption V:

$$I = I_0 e^{-V\rho} \tag{9}$$

where I_0 is constant with units of W/m^2[2].

B. Procedure

The temperature of the cell is increased from 313 K to 387 K and the detector voltage (which is a first order polynomial function of the light intensity I) is measured at approximately 2 K increments. The array of measured temperatures is converted to an array of densities using a piecewise linear interpolant of the table given above. The detector voltage at each density is depicted in Fig. 6.

C. Results

As can be seen in Fig. 6, the data is fitted to an equation of the form $y = a\exp(-Vx) + y_0$:

$$a = 4.6 \pm 0.1 \text{ V} \tag{10}$$

$$V = 3.0 \pm 0.2 \ \mu\text{m}^3 \tag{11}$$

$$y_0 = 1.17 \pm 0.06 \text{ V} \tag{12}$$

The length of the cell is known to be $l = 2.5$ cm. Assuming that the absorption of a photon is equally probable to be absorbed at any length along the cell, the cross-section area of absorption is $V/l = 120 \pm 8$ nm^2.

IV. CONCLUSION

A. Nuclear Spin Measurement

When circular-polarized light drives the D_1 transition from the ground state to the first excited state, Zeeman transitions necessarily occur. In this experiment, it is not determined whether the circular-polarized incident light is RCP or LCP. Since M_F is bounded above (by F) and bellow (by $-F$), it does not matter whether the Rb atoms are pumped to the minimum or maximum value of M_F; both states are dark and cannot absorb an incoming photon.

As calculated in Sec. II C, the value of I for ^{85}Rb is 5/2, and the value of I for ^{87}Rb is 3/2. The theoretical value of g_F is in the error bounds of the calculated value for ^{87}Rb, but this is not true for ^{85}Rb. The uncertainty in the calculation of g_F is underestimated because the magnetic field strength is assumed to be proportional to the current through the generating coils and the proportionality constant is assumed to be known exactly. In reality, this proportionality constant can potentially introduce a 5% error, which makes the measured values of g_F 0.50 ± 0.03 for ^{87}Rb, and 0.34 ± 0.02 for ^{85}Rb. These error bounds allow for the theoretical values of g_F for both isotopes.

B. Effect of Cell Temperature on Absorption

The cross-section area of atomic absorption is determined to be 120 ± 8 nm^2. The true value of the cross-section area of absorption is highly contested. By the same experimental procedure, the literature[2] arrives at 160 nm^2. By two other experimental procedures[2], the cross-section area of absorption is determined to be 1000 nm^2 and 1500 nm^2. It is likely that much of the error results from the assumption that the absorption is uniform along the length of the Rb cell.

V. REFERENCES

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[2] *Optical Pumping of Rubidium: Guide to the Experiment INSTRUCTOR'S MANUAL*, TeachSpin, Inc. (2002).
[3] Daniel A. Steck, *Rubidium 85 D Line Data*, available online at http://steck.us/alkalidata (revision 2.1.6, 20 September 2013).
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